The influence of the $4n^2$ factor on solar cells

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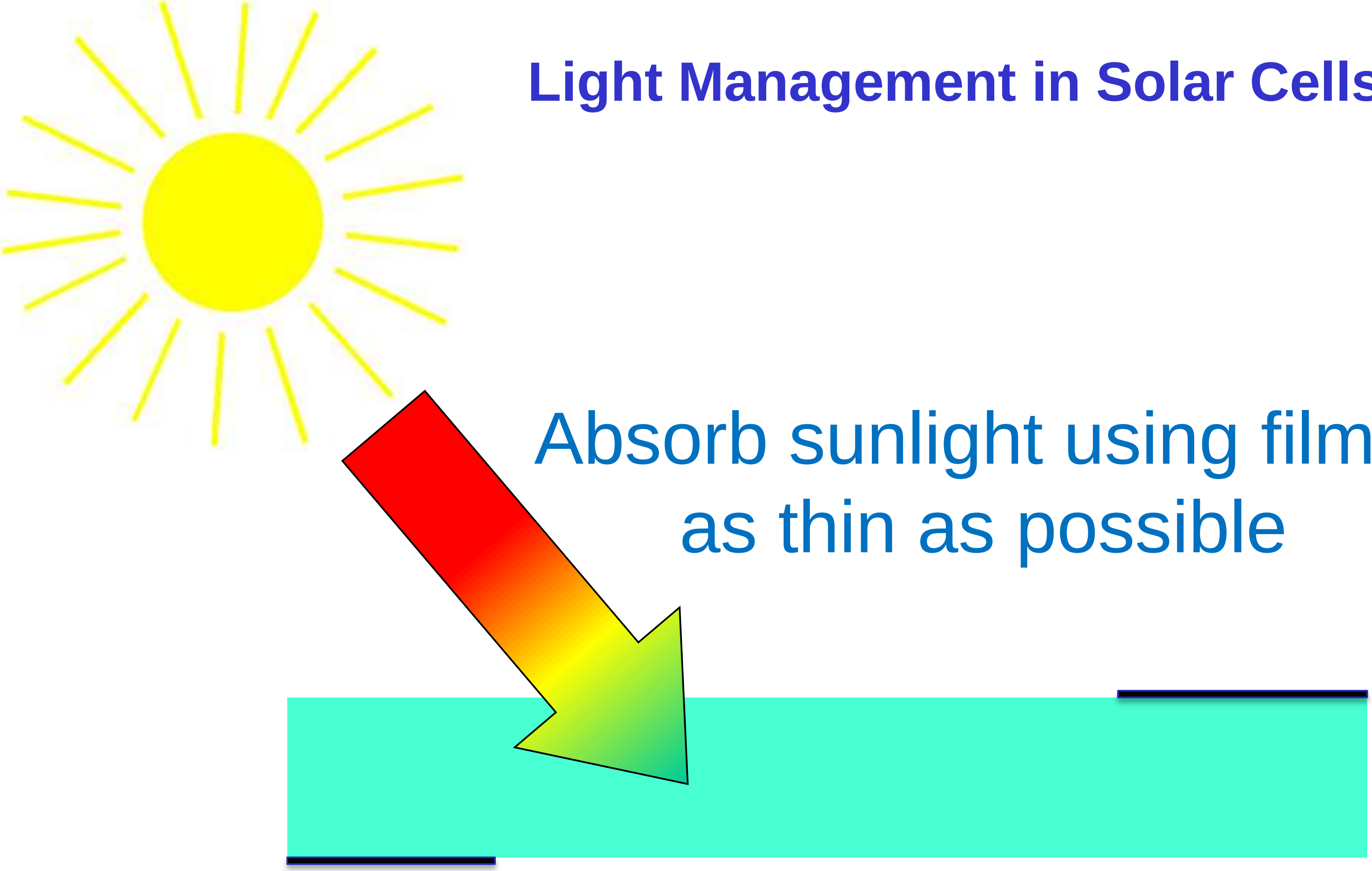
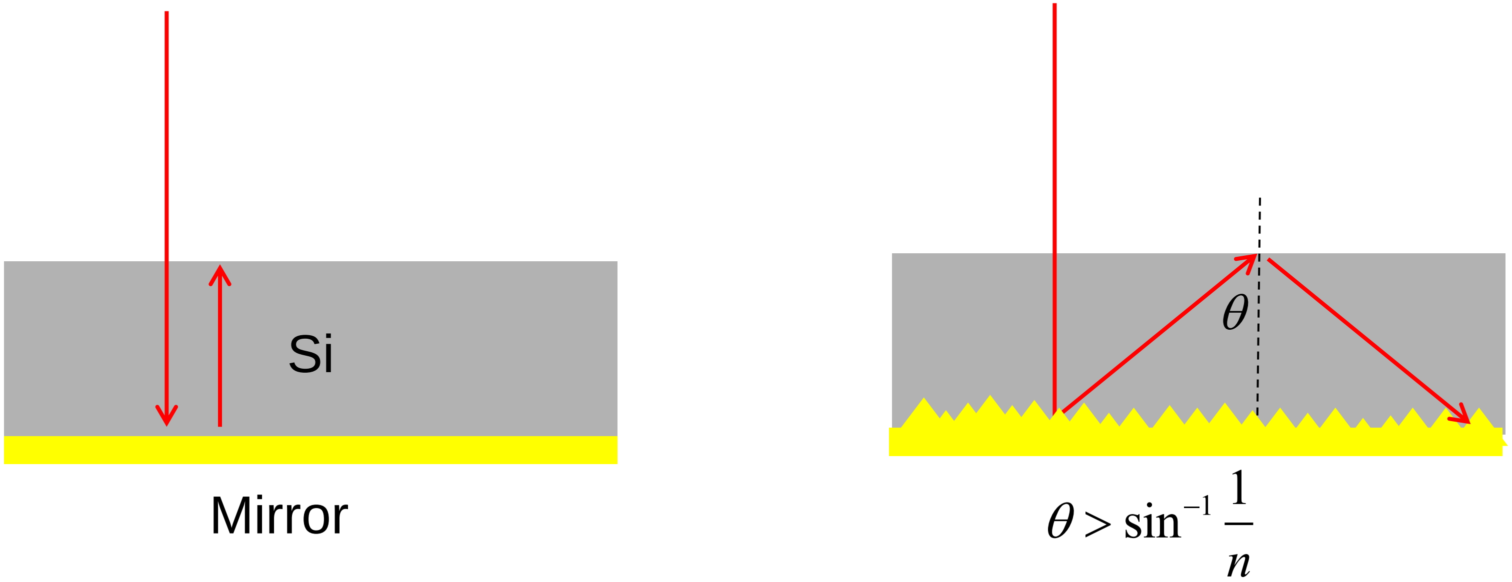
Light Management in Solar Cells



Absorb sunlight using films as thin as possible

1. **Reduce cost for expensive materials**
2. **Facilitate carriers extraction to improve efficiency**

Light trapping by roughening the surface

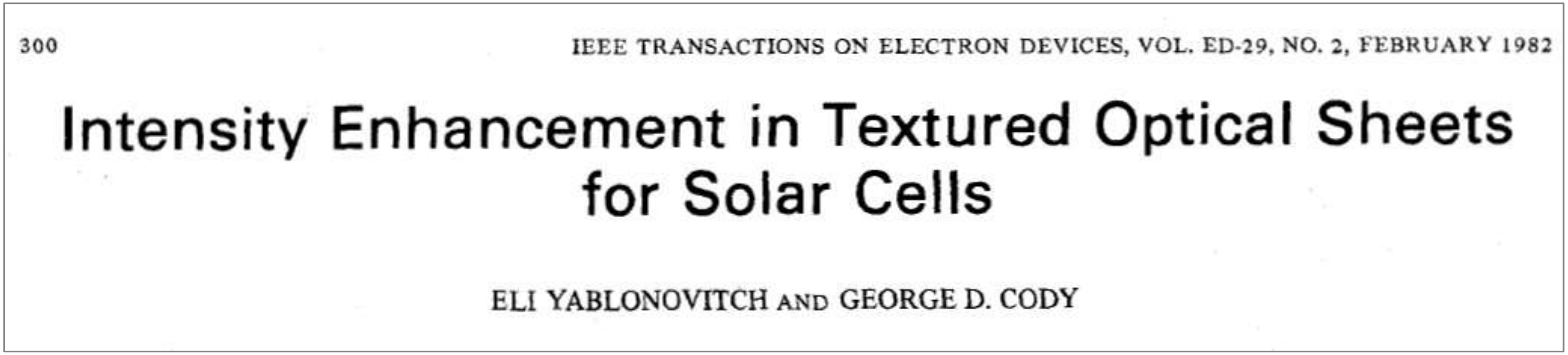


$$\theta > \sin^{-1}\frac{1}{n}$$

E. Yablonovitch, J. Opt. Soc. Am. 72, 899 (1982); Goetzberger, IEEE Photovoltaic Specialists Conference, p. 867 (1981).

$4n^2$ limit: fundamental limit of light trapping enhancement in bulk cells

Maximum absorption enhancement factor is $4n^2$ ~ 50 for crystalline silicon



300 IEEE TRANSACTIONS ON ELECTRON DEVICES, VOL. ED-29, NO. 2, FEBRUARY 1982

Intensity Enhancement in Textured Optical Sheets for Solar Cells

ELI YABLONOVITCH AND GEORGE D. CODY

Eli Yablonovitch Vol. 72, No. 7/July 1982/J. Opt. Soc. Am. 899

Statistical ray optics

Eli Yablonovitch

Thermodynamics of Light

$I_{int} = n^2 I_{ext}$

External: vacuum,
temperature T

Internal: Index n,
temperature T

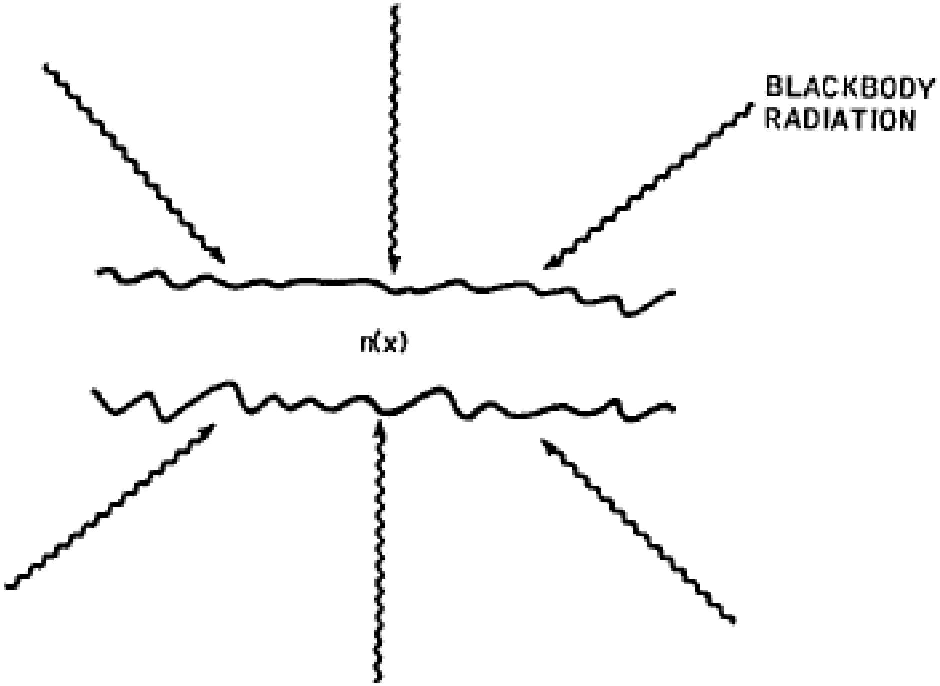
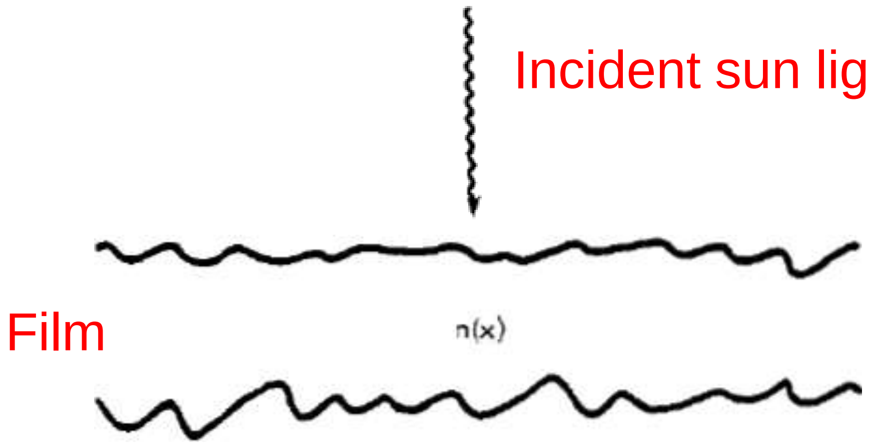


Fig. 1. Textured optical sheet bathed in blackbody radiation. The intensity inside the sheet is greater than that outside by the factor $n^2(x)$.

Yablonovitch (1982)

Ergodicity

Incident sun light is collimated



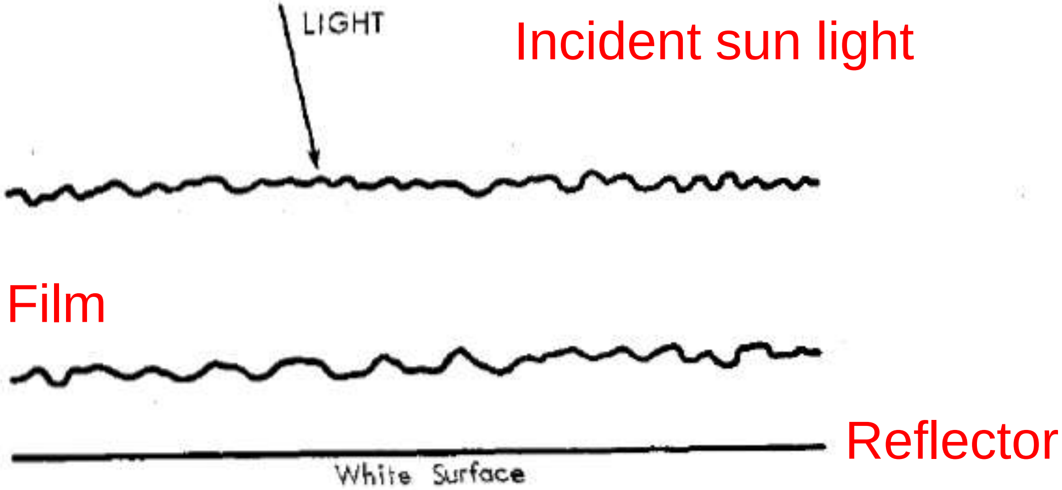
Ergodicity: once a ray enters the medium, it loses memory of where it comes from. Thus,

$$I_{int} = n^2 I_{ext}$$

should be independent of the angle of incidence.

Yablonovitch (1982)

Reflector



Incident sun light

Film

Reflector

Fig. 3. A white reflective surface effectively doubles the external intensity and increases the enhancement factor to $2n^2$.

$$I_{int} = 2n^2 I_{ext}$$

Yablonovitch (1982)

From intensity enhancement to bulk absorption enhancement

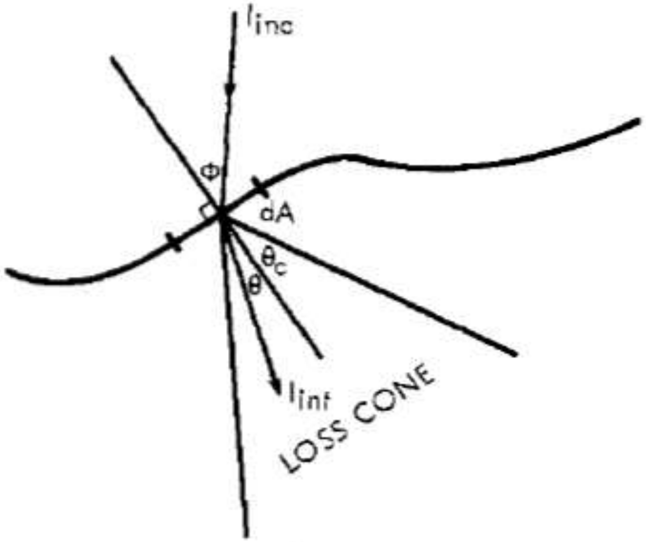
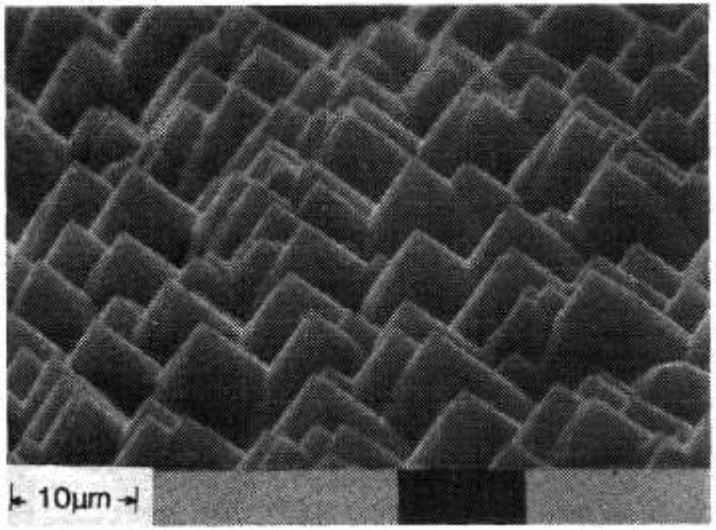


Fig. 4. The balance between incoming and outgoint radiation determines the internal intensity I_{int}.

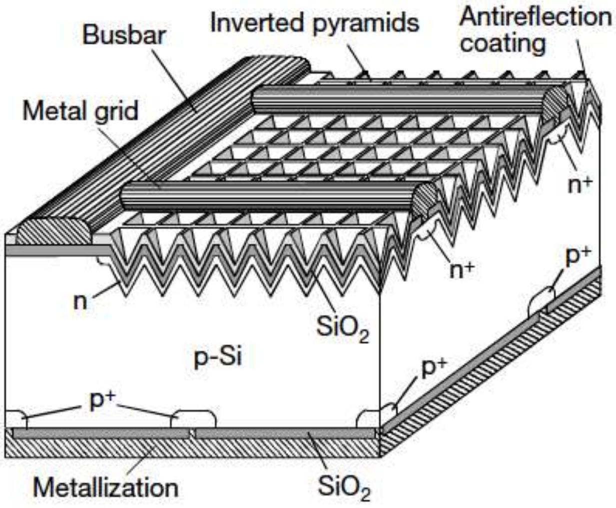
Intensity enhancement factor of **$2n^2$** leads to *bulk absorption enhancement factor* of **$4n^2$**, after takes into account the geometric factors, i.e. the light path enhancement.

Yablonovitch (1982) (With an important pre-publication correction by Dr. Swanson)

Optical design of crystalline silicon solar cells



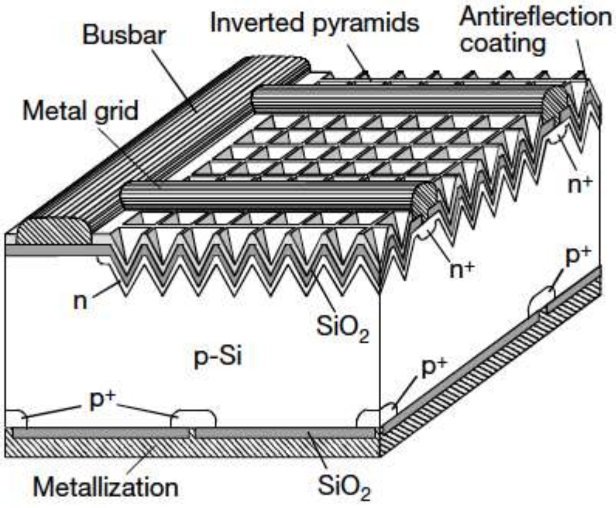
P. A. Campbell and M. A. Green, Journal of Applied Physics 62, 243 (1987)



M. A. Green, (2001)

From bulk solar cell to nanophotonic solar cells

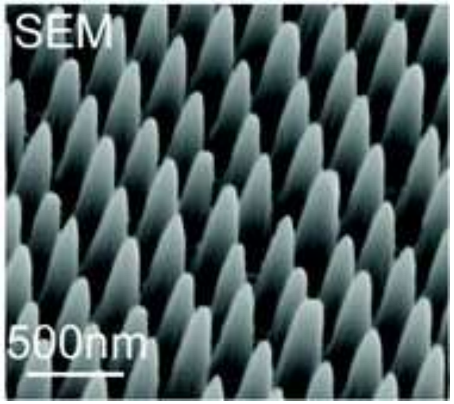
M. Green (2001)



~ 50 μm

Ray tracing

J. Zhu, Z. Yu, et al, *Nano Letters* 9, 279 (2009).



500 nm

Wave effect is important

Optical density of state is important in understanding light trapping

$I_{int} = n^2 I_{ext}$

External: vacuum,
temperature T

Internal: Index n,
temperature T

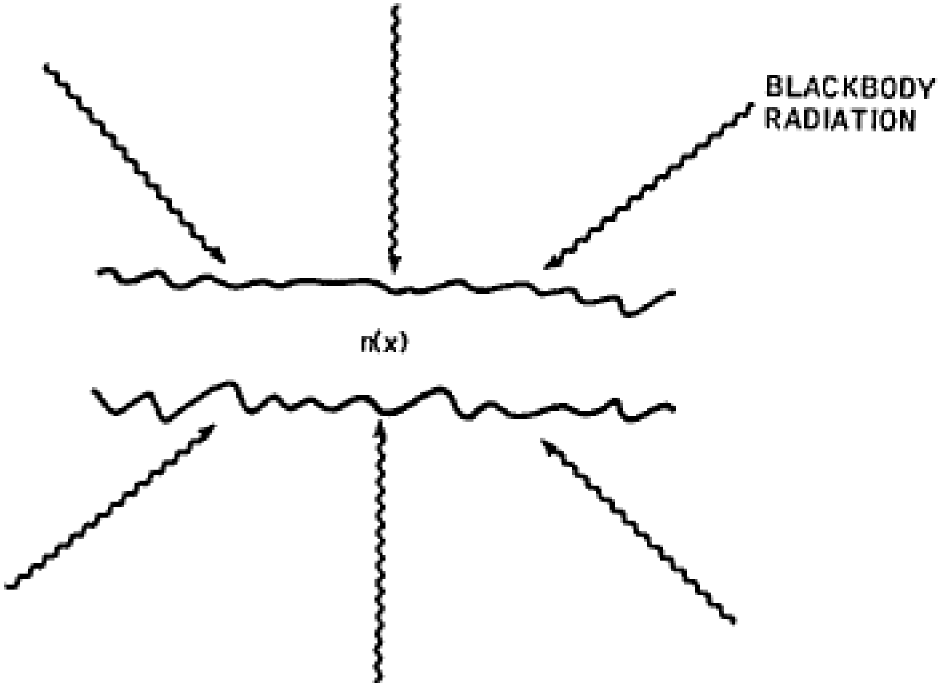


Fig. 1. Textured optical sheet bathed in blackbody radiation. The intensity inside the sheet is greater than that outside by the factor $n^2(x)$.

Blackbody radiation inside a medium is related to the density of states inside the medium

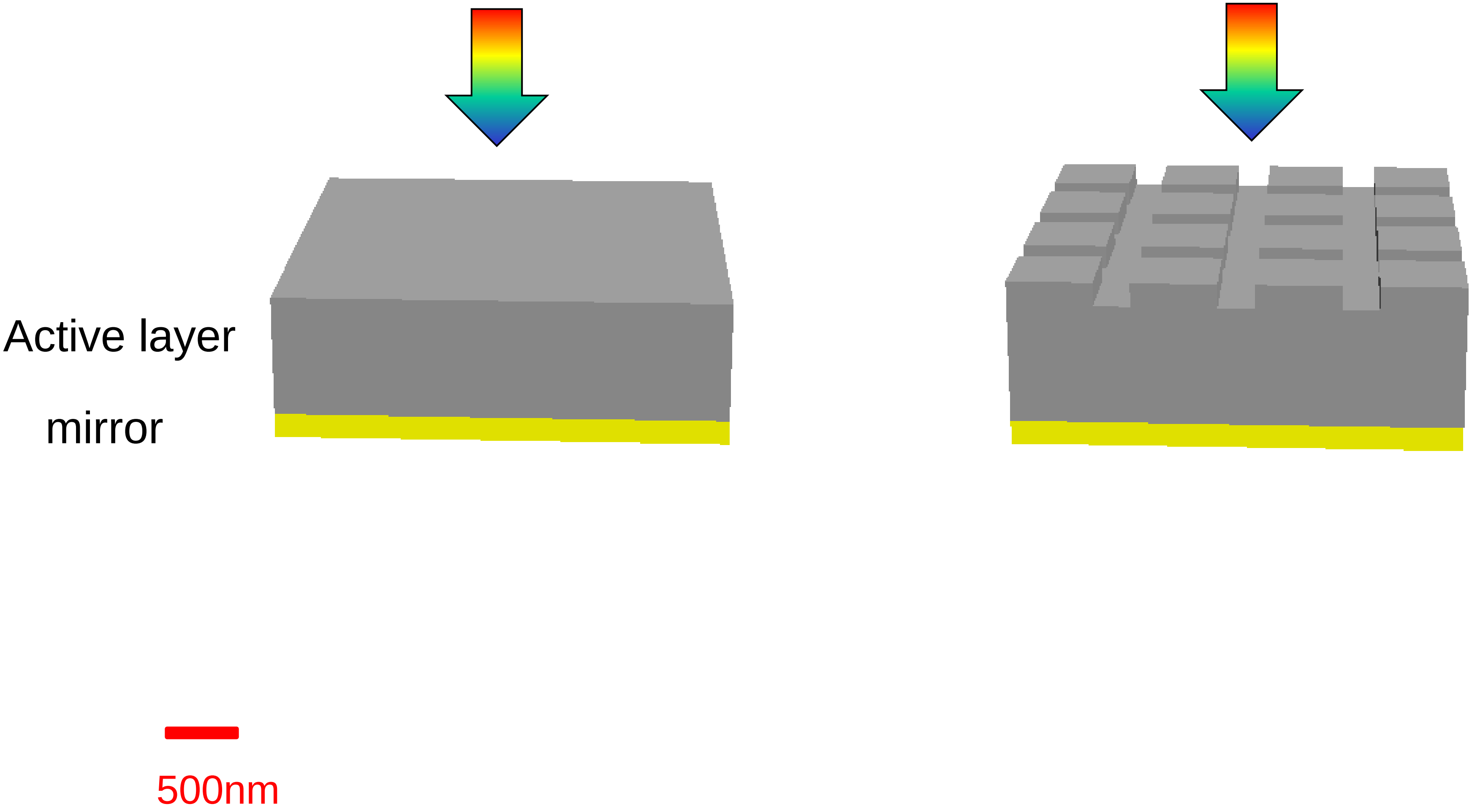
Optical density of state is important in understanding light trapping

- A density of state that is different from bulk should lead to a very different light trapping limit.
  - *P. Sheng et al, Applied Physics Letters 43, 579 (1983).*

- From the density of state perspective, light trapping in a thin film waveguide was considered in
  - *H. R. Stuart and D. G. Hall, J. Opt. Soc. Am. A 14, 3001 (1997).*

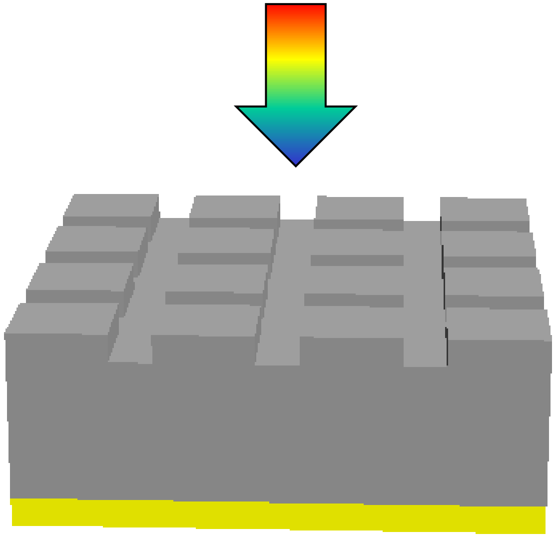
- Our own recent work, which constructs a formalism that describes light trapping entirely in terms of optical modes, without using any ray tracing concept:
  - *Z. Yu, A. Raman and S. Fan, PNAS 107, 17491 (2010).*
  - *and related work from groups including Martin Green, Harry Atwater, and many others.*

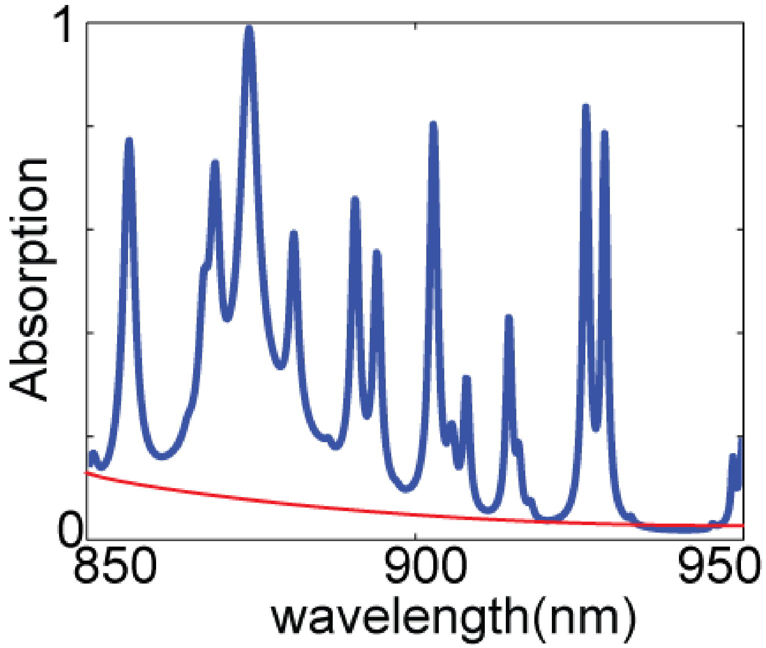
Light Trapping With Grating



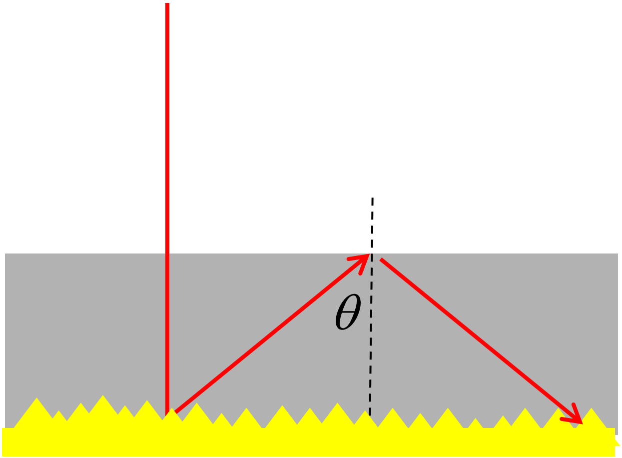
Absorption enhanced by guided resonance

- Guided resonance peak.
- Narrow spectral width for each peak.
- Requires aggregate contribution of large number of resonances.

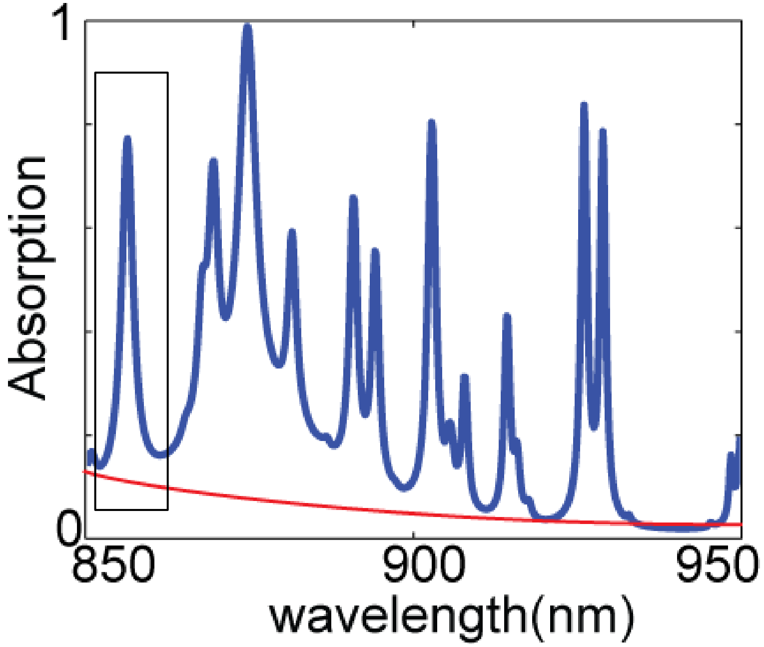




Statistical Temporal Coupled Mode Theory



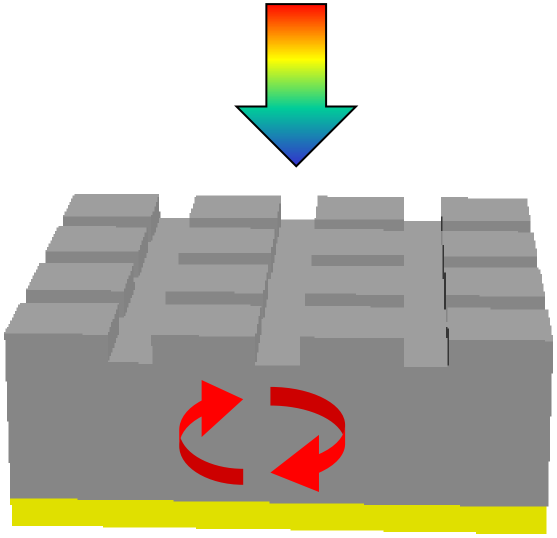
Instead of thinking about rays

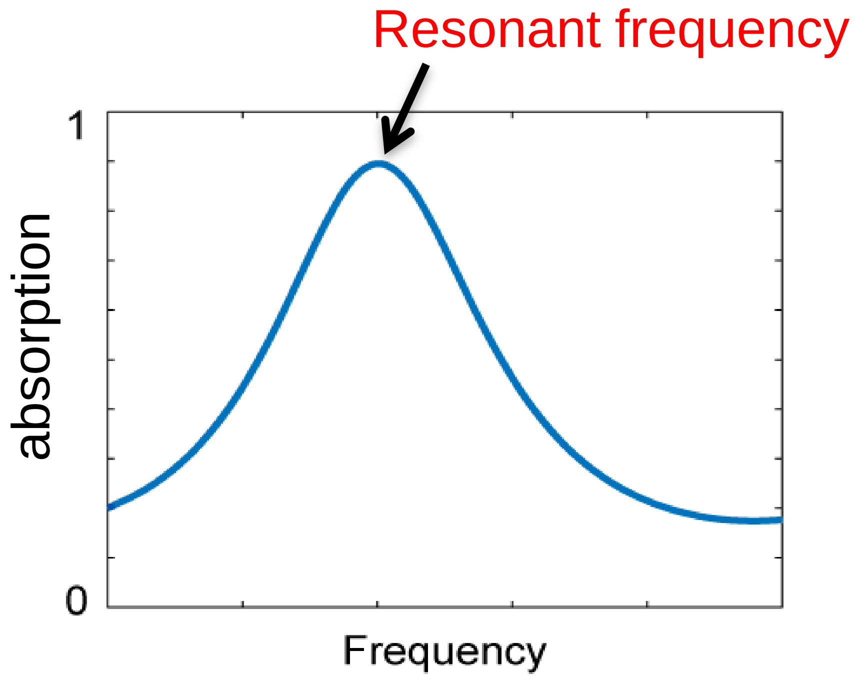


Think about many resonances

Zongfu Yu, Aaswath Raman, and Shanhui Fan
Proceedings of the National Academy of Sciences 107,17491 (2010).

A single resonance

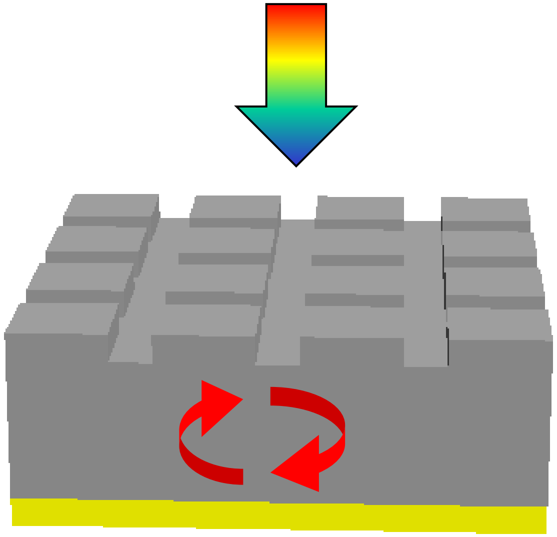


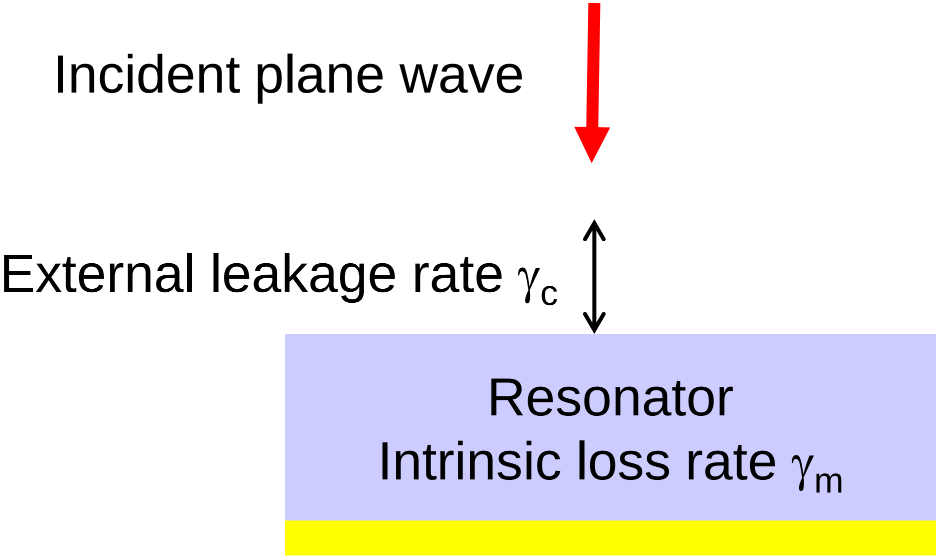


Resonant frequency

A simple model of a single resonance

Assume no diffraction in free space





Incident plane wave

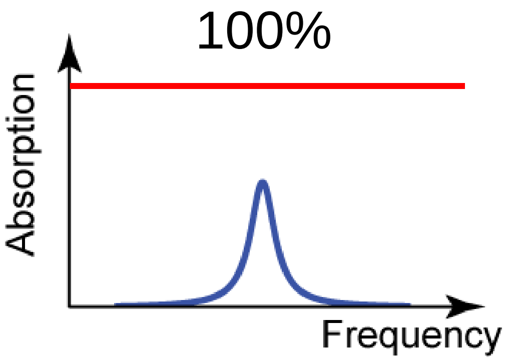
External leakage rate γ_c

Resonator

Intrinsic loss rate γ_m

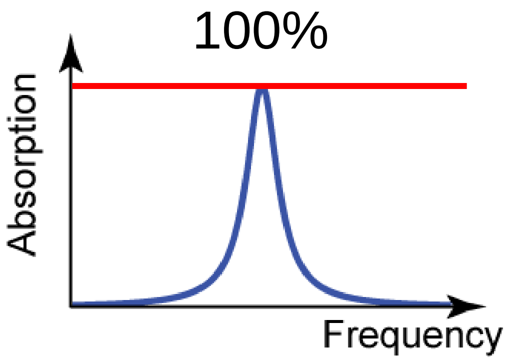
$$\gamma_m = \alpha \frac{c}{n}$$

Under, critical, and over coupling



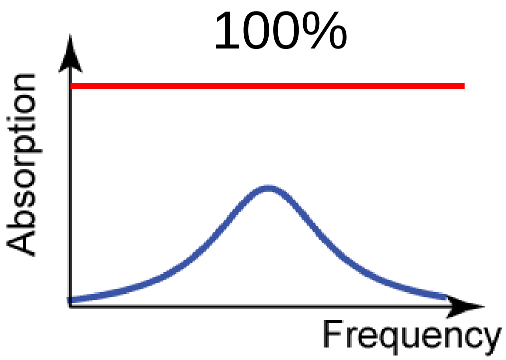
under-coupling

$\gamma_c << \gamma_m$



critical-coupling

$\gamma_c = \gamma_m$

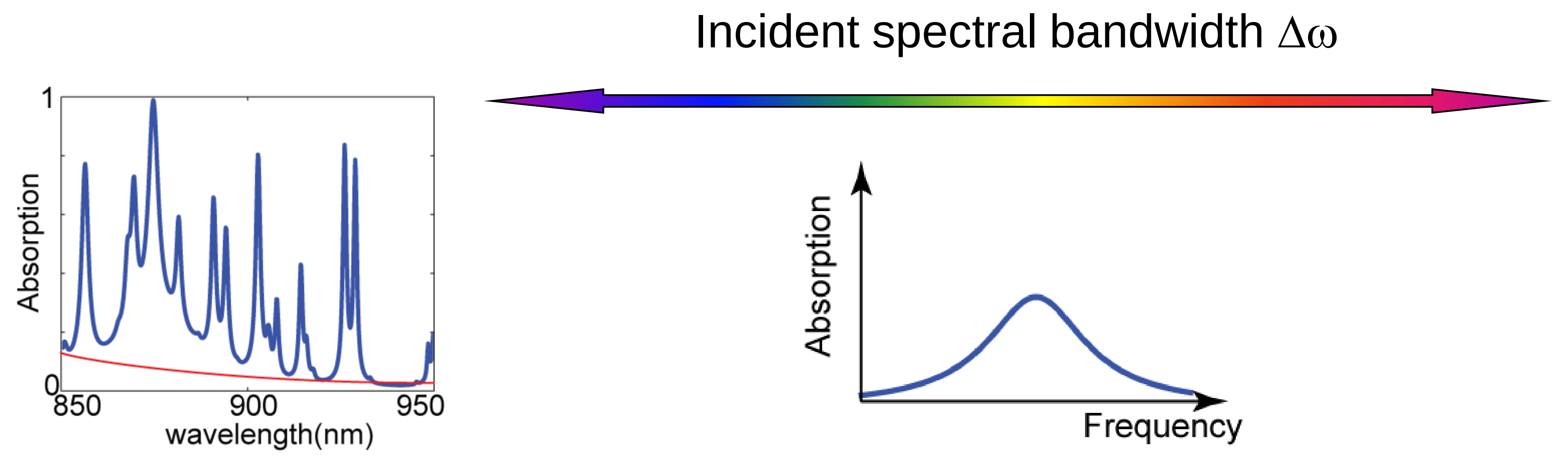


over-coupling

$\gamma_c >> \gamma_m$

Traditional use of resonance for absorption enhancement uses critical coupling

Spectral cross-section


Incident spectral bandwidth Δω

Spectral cross-section

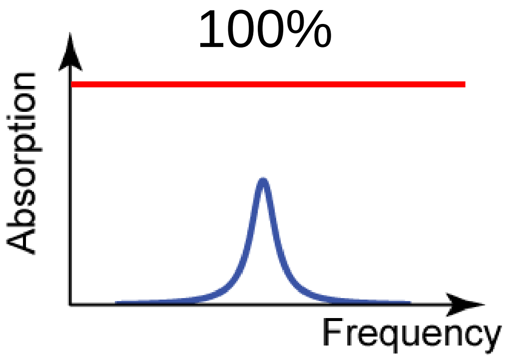
$$\sigma = \int_{-\infty}^{\infty} A(\omega)\, d\omega$$

Contribution of a single resonance to the average absorption over the bandwidth Δω

$$\frac{\sigma}{\Delta\omega}$$

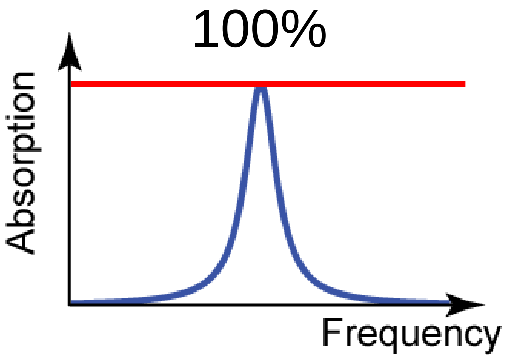
Maximum spectral cross-section

$$\sigma = \int_{-\infty}^{\infty} A(\omega) d\omega$$



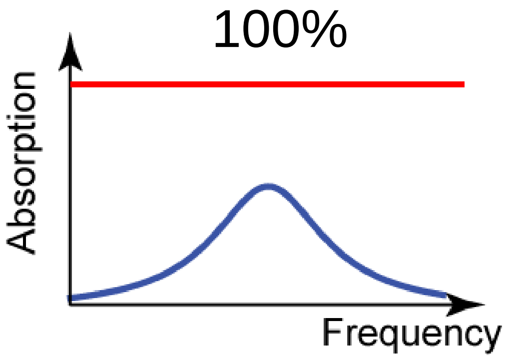
under-coupling

$$\gamma_c << \gamma_m$$



critical-coupling

$$\gamma_c = \gamma_m$$

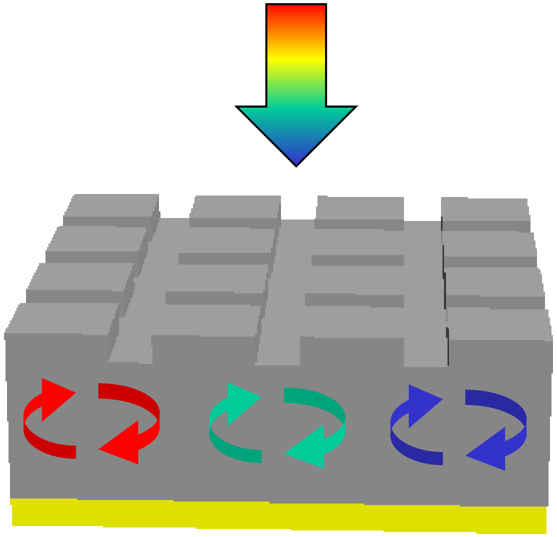


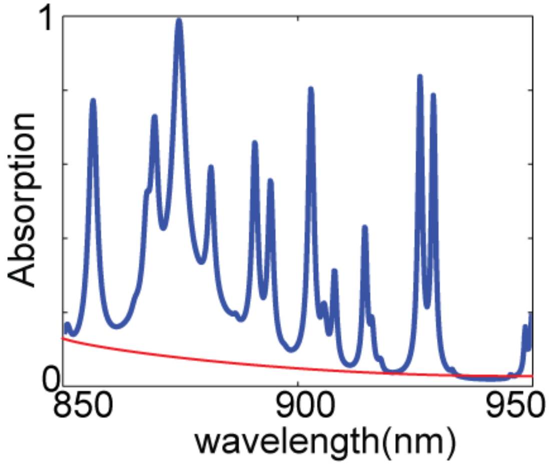
over-coupling

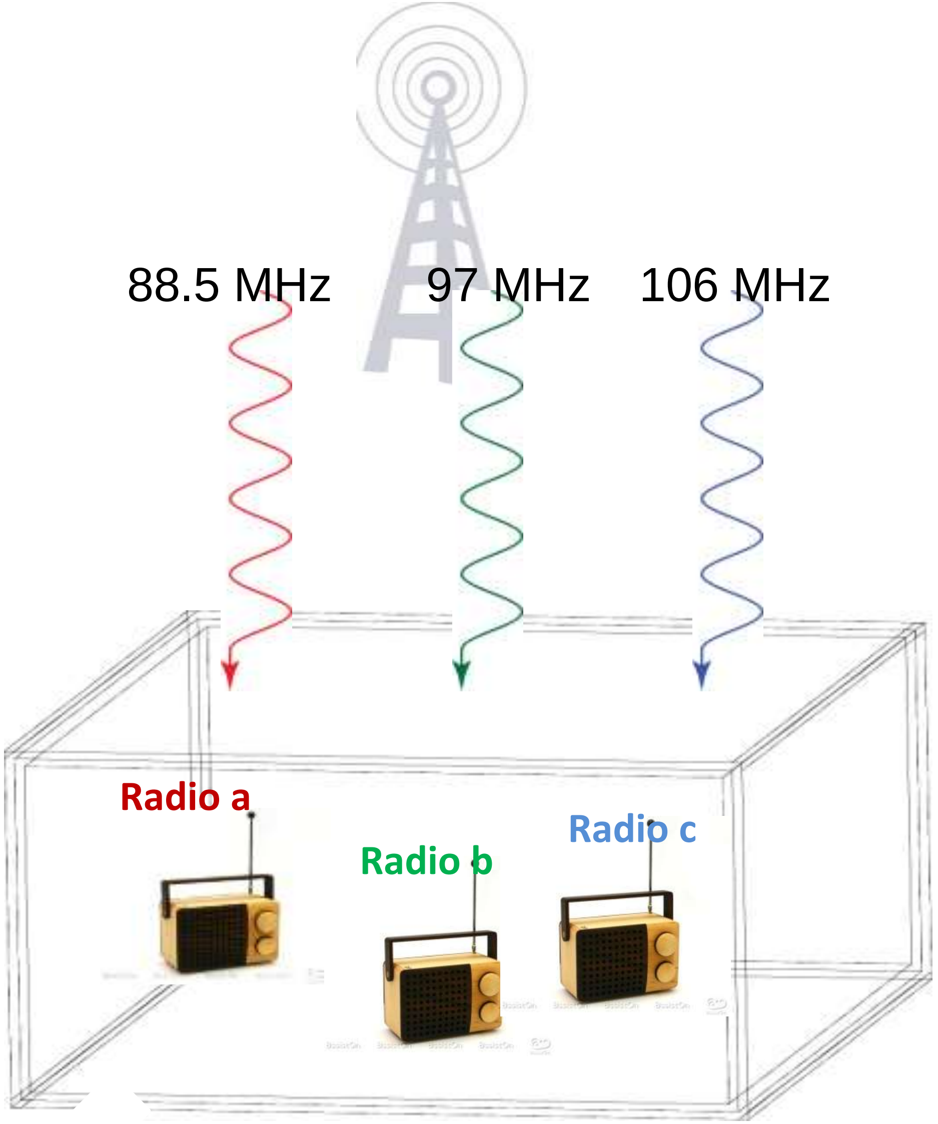
$$\gamma_c >> \gamma_m$$

$\sigma_{MAX} = 2\pi\gamma_i$ At the strong over-coupling limit, where the out-of-plane scattering dominates over the intrinsic absorption

Covering the broad solar spectrum with multiple resonances

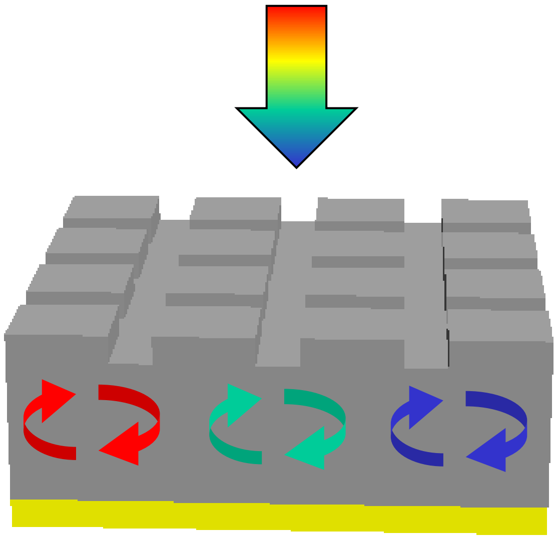


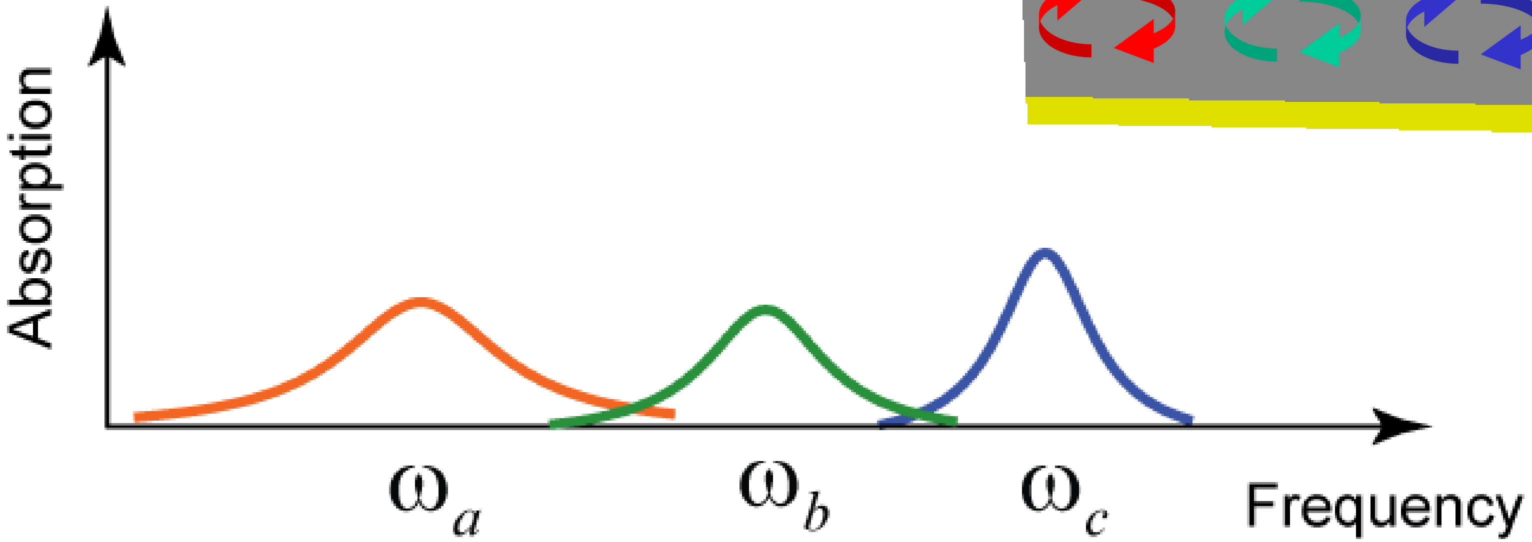




Sum over multiple resonances

M resonances





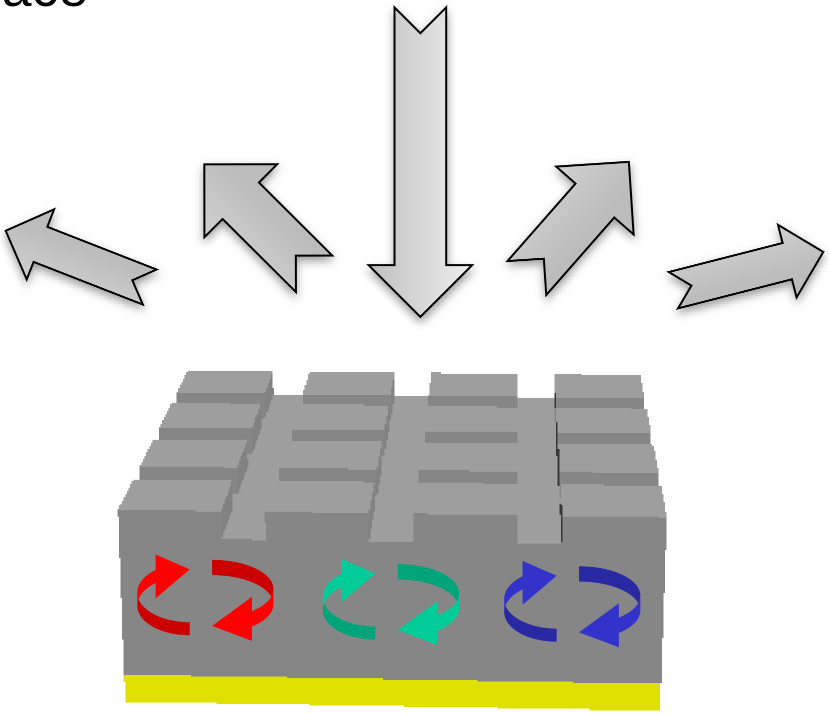
$$\sigma = \sum_m \sigma_m$$

Multiple plane channels in free space

Take into account diffraction in free space

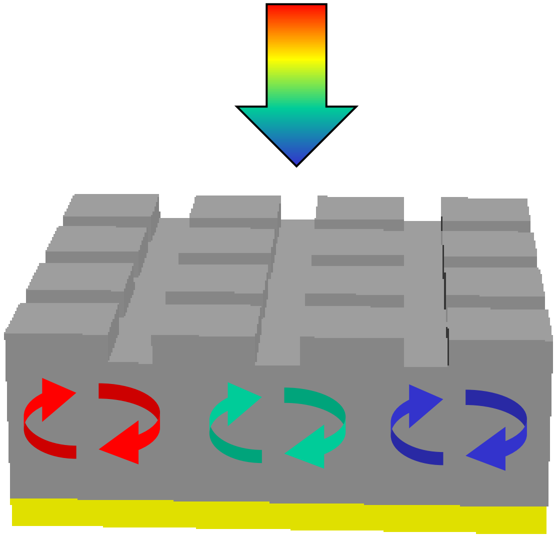
N channels

M resonances



$$\sigma_{\max} = \frac{M}{N} \cdot 2\pi\gamma_i$$

Theory for nanophotonic light trapping



Number of plane wave channels in free space: N

Number of resonances in the structure: M

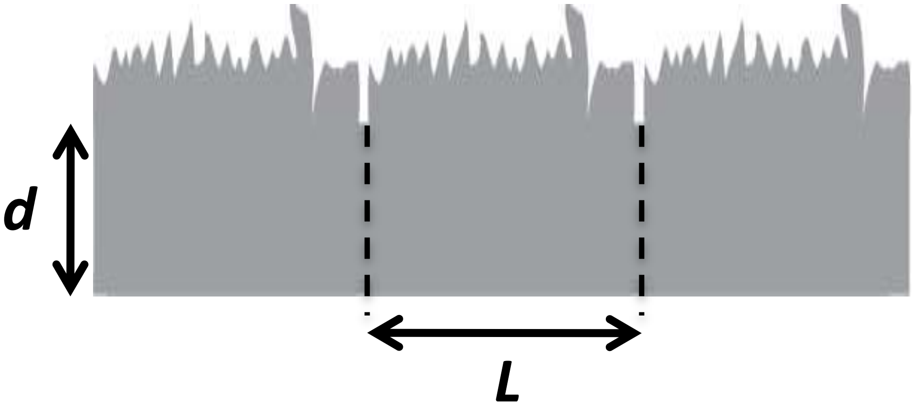
$$\boxed{\sigma = \frac{M}{N} 2\pi\gamma_i}$$

Maximum absorption over a particular bandwidth $\Delta\omega$

$$\boxed{\frac{\sigma}{\Delta\omega} = \frac{M}{N\Delta\omega} 2\pi\gamma_i}$$

Zongfu Yu, Aaswath Raman, and Shanhui Fan
Proceedings of the National Academy of Sciences 107,17491 (2010).

Reproducing the Yablonovitch Limit (the math)



Random texture can be understood in terms of grating with large periodicity

Conventional limit
Large Periodicity L >> λ
Large Thickness d >> λ

Maximum absorption

$$\boxed{\frac{\sigma}{\Delta\omega} = \frac{M}{N\Delta\omega}2\pi\gamma_i}$$

$\longrightarrow$

Maximum enhancement factor

$$\boxed{F = \frac{\sigma}{\Delta\omega} / \left(\alpha d\right) = 4n^2}$$

The intuition about the Yablonovitch limit from the wave picture

$$F \propto \frac{M}{Nd}$$

M ← Number of resonance in the film

d ← Thickness of the film

When the thickness $d >> \lambda$

Double the thickness doubles the number of the resonances

The key in overcoming the Yablonovitch limit

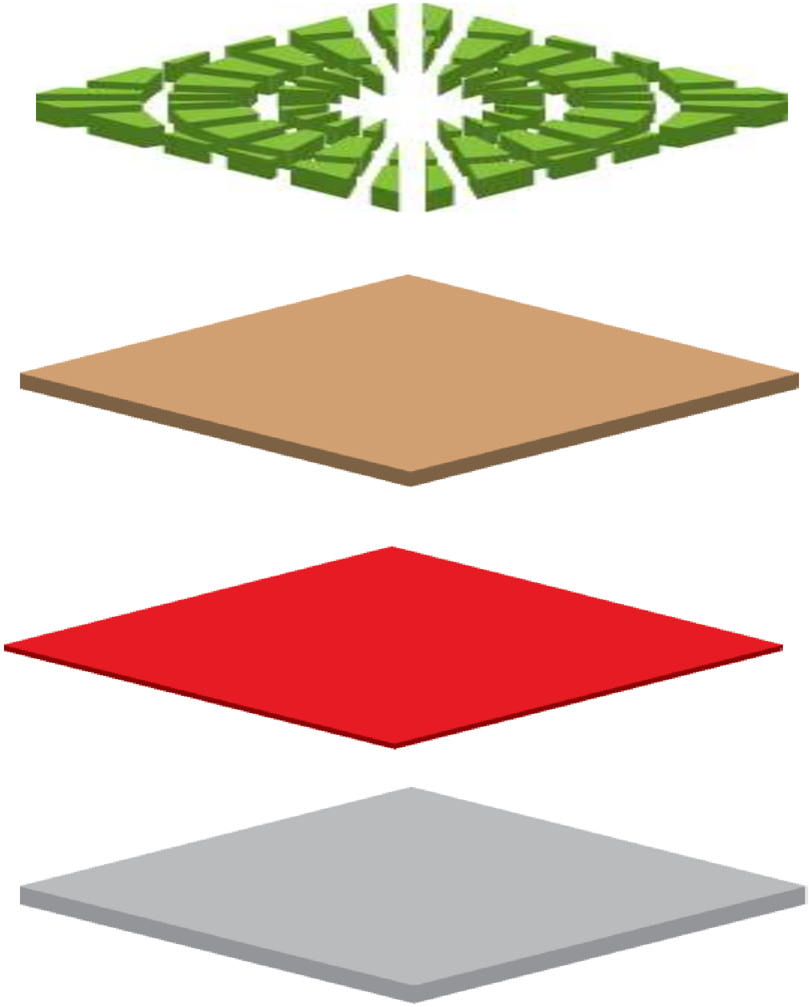
$$F \propto \frac{M}{Nd}$$

Number of resonance in the film

Thickness of the film

Nanoscale modal confinement over broad-bandwidth

Light confinement in nanoscale layers



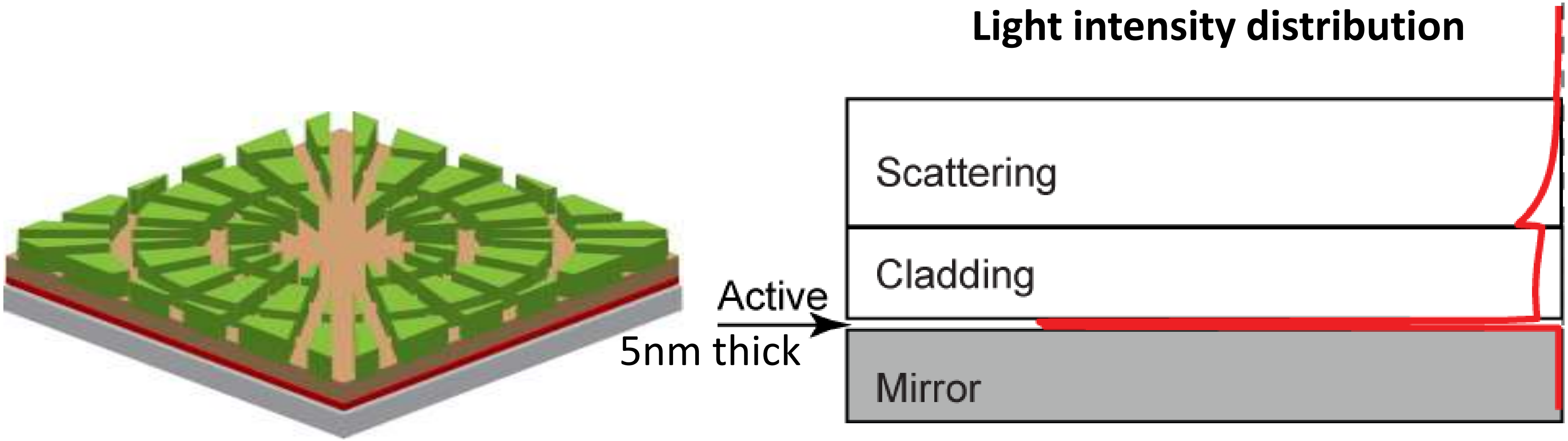
Light Scattering Layer
$\varepsilon = 12.5$, $t = 80\,\text{nm}$

Light Confining Layer
$\varepsilon = 12.5$, $t = 60\,\text{nm}$

Light Absorption Layer
$\varepsilon = 2.5$, $\alpha = 400\ \text{cm}^{-1}$, $t = 5\,\text{nm}$

Mirror

Enhancement: 15 times the classical limit



Light intensity distribution

Scattering

Cladding

Active

5nm thick

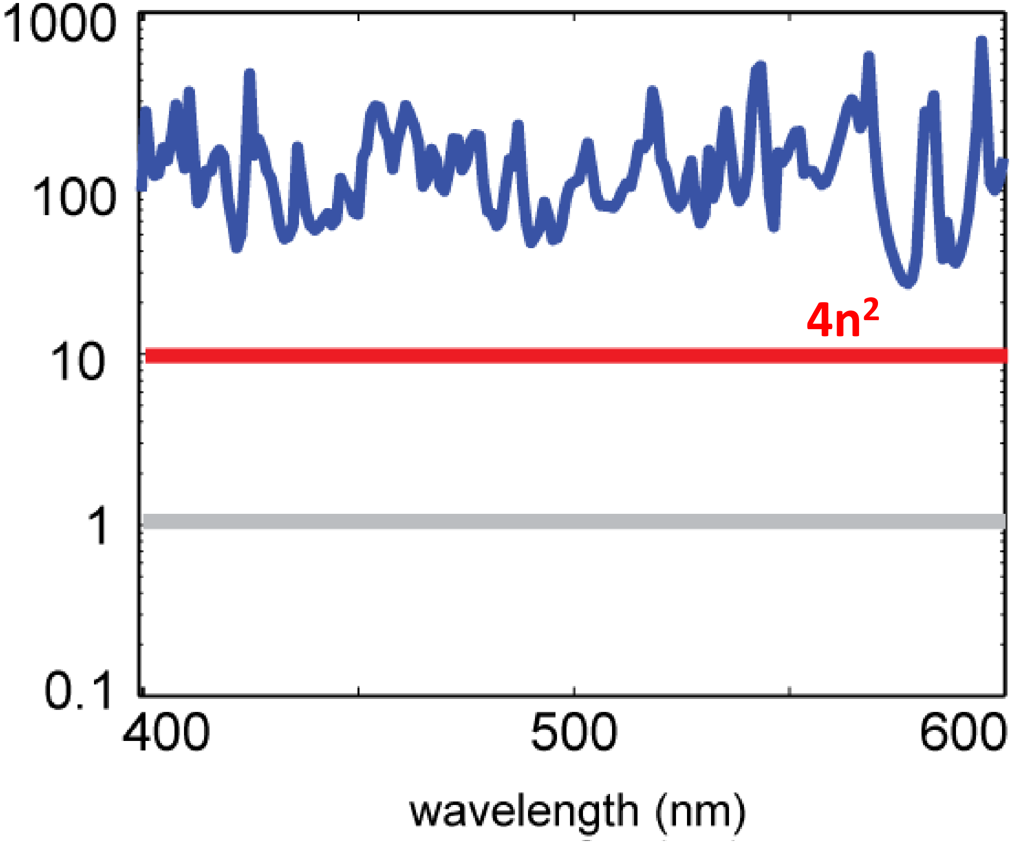
Mirror

$60n^2$

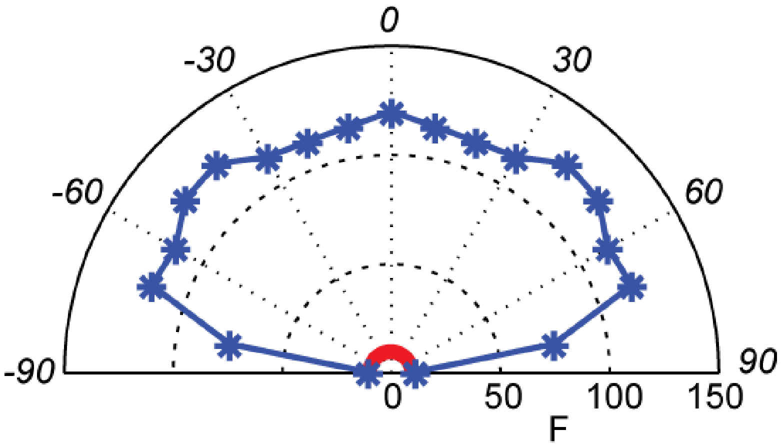
15 times of the classical limit

Simulated Absorption Spectrum

Enhancement Factor



Angular Response



Red: $4n^2$ limit

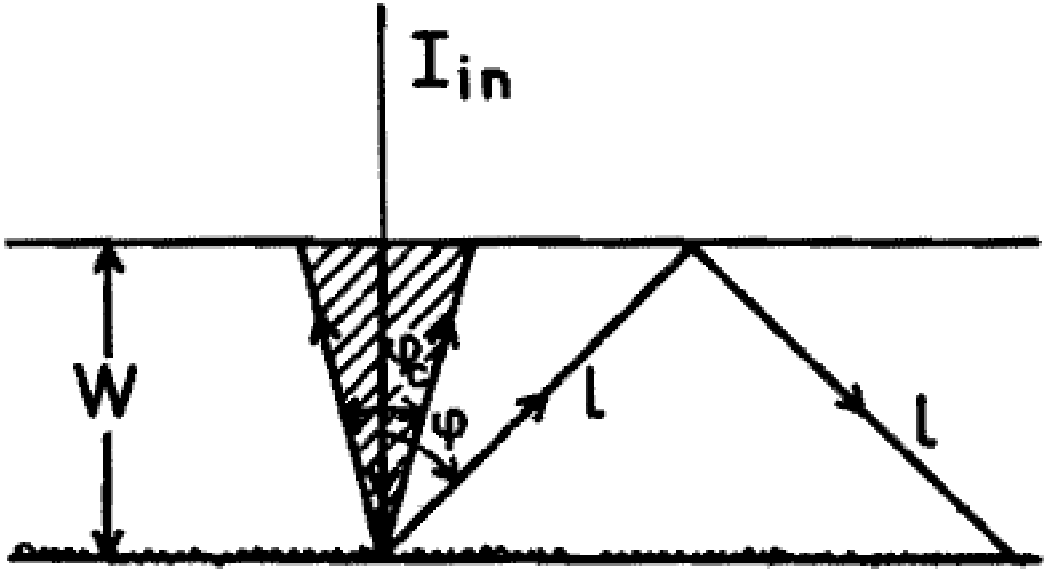
Zongfu Yu, Aaswath Raman, and Shanhui Fan
Proceedings of the National Academy of Sciences 107,17491 (2010).

Summary

- The discovery of the $4n^2$ limit has had great influence in optical solar cell designs.

- The thermodynamic understanding of the $4n^2$ limit has inspired many current works on nanophotonic solar cells.

Light Trapping



Si film

Diffuse back reflector

Fig. 2: Diffuse back reflector leads to loss cone determined by critical angle for total internal reflection φ_c.

A. Goetzberger, “Optical confinement in thin Si-solar cells by diffuse back reflectors”, Proc. of the 15th IEEE Photovoltaic Specialists Conference, p. 867 (1981).